Well-defined versus ill-defined design problem solving: the use of visual analogy

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Abstract

Analogical reasoning is considered to be an efficient heuristic for solving non-routine problems, and particularly helpful in design. It is during the design process, where a large collection of visual displays aid designers, in which the use of visual analogy is of specific importance. Few works have studied the effect of the use of visual analogy in design problem solving, and there is no research which has studied whether it plays a more significant role in the context of ill-defined problems or in well-defined problems. The objective of this study is to empirically compare and investigate the use of visual analogy in well-defined design problems (routine) and ill-defined design problems (non-routine). Results showed that students benefited from the use of visual analogy, which significantly helped them to improve design solutions in both design contexts. Additional results showed that architects also benefited from the use of visual analogy in ill-defined design problem solving. In contrast, visual analogy did not aid them to improve their performance in solving well-defined design problems.

Well-defined versus ill-defined design problem solving: the use of visual analogy

Well-defined and ill-defined problems

A main distinction has been established, between *well-defined problems* and *ill-defined problems* (e.g Gero and Maher, 1993; Goel, 1995; Medin and Ross, 1990; Mitchell, 1993; Reitman, 1964; Rittel and Melving, 1984; Simon, 1984). Well-defined problems are defined by: completely specified initial conditions, clear goals, a defined set of operators for transforming conditions, and a limited number of solutions. A classical example of a well-defined problem is a sum problem, because it meets all the necessary requirements. Well-defined problems are called routine when they allow the use of efficient algorithms to generate solutions that may fully satisfy the initial requirements (e.g., Cross, 2000; Visser, 1996). Ill-defined problems, on the other hand, can be defined by: no clear initial conditions, no completely specified goals, a large number of unpredictable solutions, and no defined set of operators or algorithms. Since solutions to ill-defined problems may be ambiguous, it is not possible to forecast whether an algorithm may fit the initial requirements. For this reason, ill-defined problems cannot be solved in a routine way. These types of problems are associated with the generation of different novel solutions to a similar problem situation. A main feature associated with non-routine problems is the generation of unexpected solutions that are significantly different from prior problem situations (e.g. Suwa et al, 1999). Since this is often the case, design problems are generally considered as prime examples of ill-defined and non-routine problems.

Design problems as main examples of ill-defined problems

Design problems are usually considered as fundamental examples of ill-defined problems (e.g., Gero and Maher, 1993; Goel, 1995). Design methodologists who studied problem solving directed their attention to well-defined problems (e.g., Jones, 1970; Lawson, 1980), and routine processes. They did not understand the ill-structured nature of *design problems*, and thought that design could be studied as well-structured problem-solving. By considering the design process as a series of connected and sequential well-defined steps, the methodologists proposed that rational prescriptive models of design problem-solving, might aid in finding optimal solutions. However, instead of helping to gain a deep insight into the design process, these models over-simplified rich and complex aspects of design.

Recent research has shown an increasing interest in studying ill-defined design problems. A main feature of these design problems is, the generation of solutions that have no obvious relation with, or that are considerably different from prior existing design solutions (e.g., Goldschmidt, 1994; Suwa et al, 1999). This suggests that in ill-defined problem solving, the range of possible solutions can be extended to unknown and unexplored possibilities.

Analogy as a problem solving strategy

Analogical reasoning is considered to be an effective heuristic in dealing with problem solving, particularly with ill-defined problems where as noted above, the production of novel solutions is possible. An *analogy* is defined as a resemblance of structural relations, as in A:B :: C:D, or A is related to B like C is related to D, where D is the unknown term that has to be established. The use of analogy implies the transfer of related abstract information from a known domain (source), to a situation that should be explained (target), (e.g., Gentner, 1983; Novick, 1988; Vosniadou, 1989). Reasoning by analogy depends on the application of a system of structural relations to the problem at hand. The use of analogy is a cognitive mechanism that enables one to retrieve old information that can support the acquisition of new knowledge. The identification of a similarity between known relations in the source situation and potential relations in the target situation, allows for the creation of an analogy (Pierce and Gholson, 1994). A system of relations of relevant knowledge (common higher abstraction) is transferred from base to target by analogical mapping (Dejong, 1989).

Visual analogy in design problem solving

Research in *visual analogy* was almost absent from cognitive science. Exceptions are the recent studies achieved in visual analogy in problem solving (e.g., Antonietti, 1991; Bean et al, 1990; Beveridge and Parkins, 1987; Gick and Holyoak, 1983; Verstijnen et al, 2000). However visual analogy, as compared to analogy, was never considered an independent category. Therefore, its contribution to problem solving in general and to design problem solving in particular, was not completely appreciated. During the design process, designers constantly refer to and frequently use visual displays. These references and uses, are by themselves important reasons why visual analogy can be considered to be a helpful cognitive strategy for improving the quality of the design outputs. Goldschmidt (1994a; 1994b; 1995; 1999) who studied the use of visual analogy in design, proposed that while looking for a suitable solution the designer tries to identify clues from relevant visual displays in order to establish mappings with the design task. There are a number of anecdotal cases of well-known architects illustrating the successful use of visual analogy. For example, Le Corbusier has implemented a number of analogs into the designs of different buildings, such as ships and wine bottle-racks. The shell of a snail also served Le Corbusier as a main analogical base to design the “endless” plan of the Museum of Tokyo. Similar examples of analogical transfer from a natural phenomenon to a design instance are reported in the works of Calatrava. His ingenious structural inventions are the consequence of using animal skeletons and tree branches in his design process.

Most of the early studies in the fields of cognitive science and design directed their attention to well-defined problem solving, while only recent works have studied ill-defined problem solving. Recently, a few empirical studies have been carried out, on the use of visual analogy in ill-defined problem solving (e.g., Casakin & Goldschmidt, 1999; Casakin & Goldschmidt, 2000; Verstijnen et al, 2000). However no empirical work has focused on a comparative analysis between ill-defined and well-defined design problems.

Empirical research

Objectives and hypotheses

The objective of this empirical study is to verify possible differences in the role played by visual analogy in ill-defined and well-defined design problems. We would like to test, to what extent students and architects are able to use visual analogy, and how this contributes to enhancing the quality of their design solutions in well-defined and ill-defined problem solving.
The major hypothesis is that, the use of visual analogy will help student and professional designers to improve their performance in ill-defined design problem-solving, but will not aid them so much in well-defined problem-solving. To validate this hypothesis, a comparison is made between results obtained in solving both types of design problems in each group of subjects.

Subjects

63 architectural designers belonging to three groups with different levels of expertise participated in this experiment while solving ill-defined problems. They were divided into 17 architects, 22 advanced students, and 24 beginning students. In the well-defined context, a total of 54 architectural designers divided into 17 architects, 17 advanced students, and 20 beginning students participated in the experiment.

Experimental conditions

In order to achieve the objectives of this study, two experimental conditions in which subjects were required to solve the design problems, were implemented as follows.

Test condition: Solving design problems with visual displays, and with an explicit requirement to use analogy:

Subjects were provided with general instructions and a description of the design problem. Together with these they were given a board with an assortment of visual displays including, images from the architectural domain as well as from remote domains. They were informed that some of the images might serve as potential analogs for the design problems. The subjects were required to identify relevant sources, and to use analogy while generating solutions to the design problems they were given.

Control condition: Solving design problems with the aid of visual displays but devoid of any explicit requirement to use analogy:

A similar task was given to subjects with the same degree of design expertise, as in the previous experimental condition, consisting of the same instructions, design requirements, and visual information. However, they were not explicitly required to use analogy.

The three design problems solved in the ill-defined context were: a) the prison b) the dwellings and c) the viewing-terrace. The two design problems solved in the well-defined context were: a) the staircases and b) the parking-garage.

Procedure

The experiments were carried out in individual design sessions (one participant at a time). Subjects were provided with general instructions, and a description of the problem's requirements. They were then given approximately 20 minutes to solve the design task. At the beginning of the session the experimenter answered subjects' questions, but did not intervene throughout the duration of the experiment. It should be noted, that various subjects solved more than one design problem under the test or control condition, so therefore the number of statistical 'entries' as described below exceeds the number of subjects. However, in these cases, design tasks in the control condition were always given before design tasks were provided in the test condition.

Equipment and materials

The Research Laboratory room used for the experiments was small and soundproof. The subject was shown a 1m x 0.7m board containing a vast assortment of visual displays, which varied according to the problem at hand. The boards included an average of twenty-four images classified according to: a) pictures from the architectural design domain, to which the problems belong (within-domain sources) b) pictures from other remote domains (between-domain sources) like science, art, or engineering. Some of these images could be related to the design problem, while others could not.

Scale of assessment

An ordinal scale from 1 to 5 points was established, in order to evaluate the design solutions for ill-defined design problems. A range from 1 to 2 points was assigned, to cases where the design solution did not satisfy the design requirements, and a range from 3 to 5 points was assigned to cases where the design solution, did satisfy design requirements. A different scale, of 0 or 1 point, was established to assess solutions for well-defined design problems. Zero was assigned where the solution did not satisfy design requirements, and 1 point when the solution was seen as satisfactory.

Judges
Three naive judges unaware of the test conditions, scored the design solutions produced in the different experiments, independently. All of them were architects with at least seven years of professional experience, who volunteered their time. A reliability analysis showed a low disagreement rate among the judges for all the design solutions (average of 3%).

Statistical analysis methods
The scores assigned by the judges in the context of the well-defined design problems, were tested using Fischer Exact's Tests. The scores assigned in the context of the ill-defined design problems, were submitted to T-Tests. Differences between subject groups, were considered significant at a level of 90% ($p<0.1$). For statistical analysis considerations, the three ill-defined design problems ('the dwellings', 'the prison', and 'the viewing terrace') were grouped together. Similarly, the two well-defined design problems ('the staircases' and 'the parking-garage') were grouped together.

Well-defined and ill-defined design problems: Qualitative results
In this section we describe two individual sessions, carried out by two novice designers, while solving well-defined and ill-defined problems. The purpose is to illustrate two different cases, in which student designers successfully used visual analogy to solve the assigned design problems. In the well-defined problem session, the student dealt with the 'parking-garage' problem. The subject was required to arrange, the internal subdivision of a 15m high parking-garage building, in order to accommodate 120 cars. A 6m wide two-way passage was required for the internal circulation of cars. Two external lifts were required, to elevate cars through the different floors (See figure 1). The building was divided in two split-level wings, with one-meter difference in their respective lengths. While 60 cars could be easily allocated in the longer wing, the main design problem was to find a solution to arrange the rest of the cars in the shorter wing.

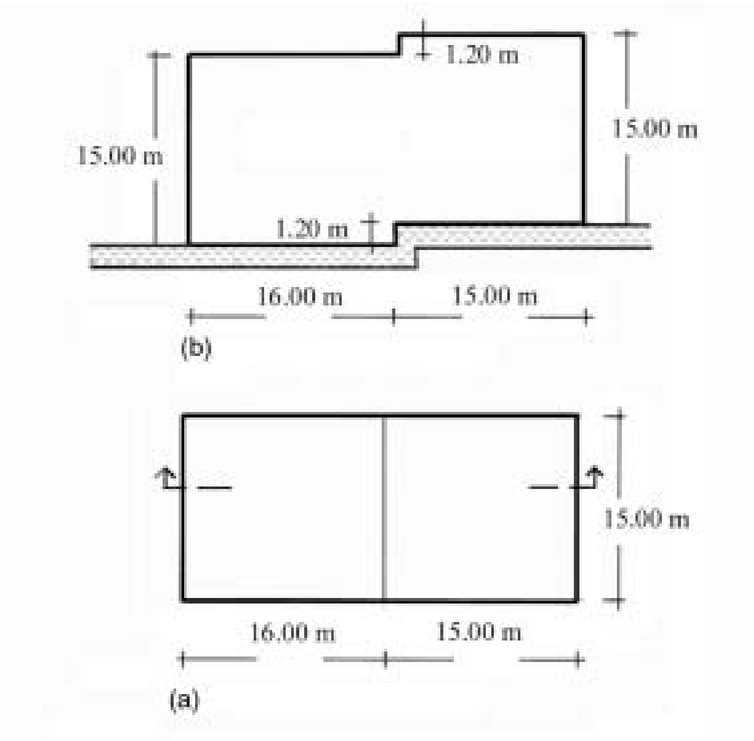


Figure 1: Plan and section drawings for the 'Parking Garage' problem.

In the ill-defined problem, the student dealt with the 'viewing terrace' problem which was comprised of the schematic design of a 30m^2 viewing terrace which had to be located at the highest point of a 16m high precipice. A main constraint was to divide the terrace into two different sectors. While one sector was required to have maximal contact with the ground, the other was required to have minimal contact with the ground. The descriptions below are based on protocols obtained from recordings of the two different design sessions.

Successful well-defined problem solving aided by visual analogy

The design session begins with the student analyzing the design problem. While focusing on the design constraints the subject says:

"The height of the building is 15m, [and it is] divided in two wings. One [wing has] about 15x16 m and the other [has] 15x15. There is a 1.20m height difference between both wings of the building. I have to arrange 120 cars… The minimum height [per floor] within the building has to be 2.60m."

In order to illustrate an understanding regarding the provided information about car arrangement within the parking-garage, the student decides to produce a first sketch and says:

"First of all I am trying to see how can I organize the interior of the building. I am going to check the way that cars relate to the corridor…[figure 2]. 5,6,5…This will be the car/corridor relationship… So let's see if there is any problem…"

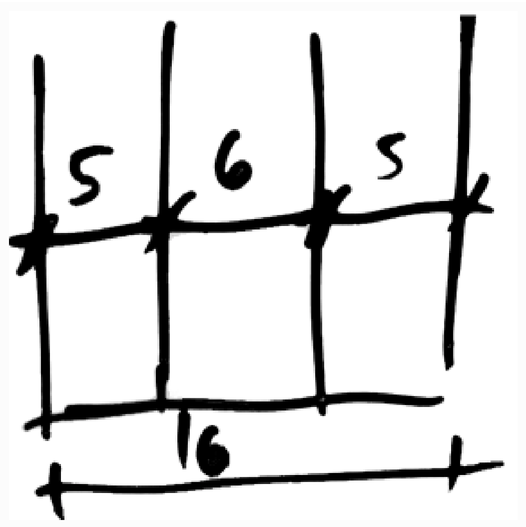
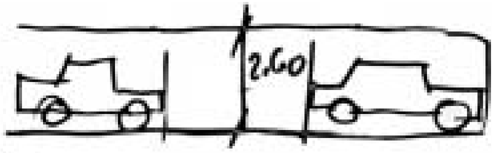


Figure 2: Sketch I (well-defined problem). Distribution of cars along corridor.

In a second step, the designer starts thinking about the possibility of representing the organization of cars in three dimensions. In order to do so, the subject manipulates the spatial arrangement of cars by referring to plan and section drawings. These lead to a realization that in order to be able to find a suitable design solution, space shortage constraints need to be included.

"Considering that each floor should have 2.80m at least … I can divide the building height [15m] into six floors…[figure 3]. Sixty cars can be easily located in one of the wings… I am going to check what can be done in the second wing. Oh… now I can see the problem… The second wing is [one meter] shorter in plan so that I do not have enough space to organize the 60 remaining cars like in the other wing."



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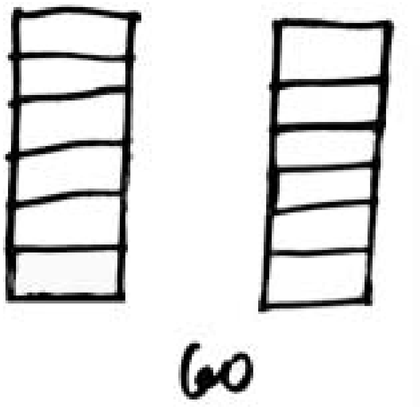
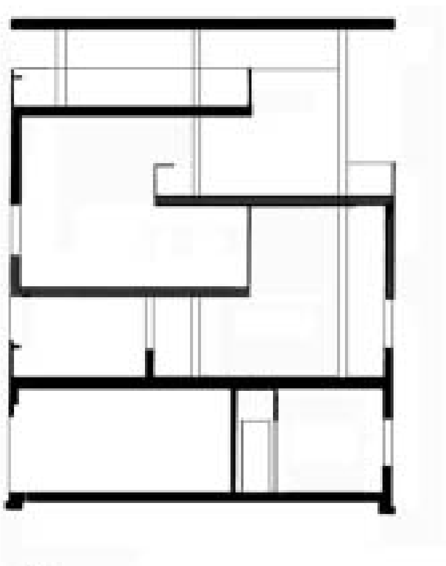


(a)

Figure 3: Sketch II (well-defined problem). Distribution of cars in (a) plan and (b) section drawings.

Only after understanding the scope of the design problem, the novice student begins to examine the assigned board containing the set of visual displays. While trying to identify similarities that can potentially serve to establish an analogy with the problem, attention is focused on a ‘within-domain’ visual display, and a ‘between-domain’ visual display. In so doing, relevant analogical principles such as ‘partial superposition’ and ‘split-level’ are discovered.

“I am now looking at the board, trying to see what can I get from the visual displays. There is a general principle dealing with organization between things… I can see that in the dwelling [figure 4a] there is something like a split-level situation [principle] between the different floors. This can match the problem of height differences [between both wings]. Probably the figure of the zigzag [figure 4b] may be of help to arrange objects, but I think that [the image of] the dwelling is clearer to me. That is the way forms [floors] superpose one with the other. I can now figure out how to deal with the problem of the split levels, and on the other hand the possibility of partial superposition [between levels to deal with space shortage]…”



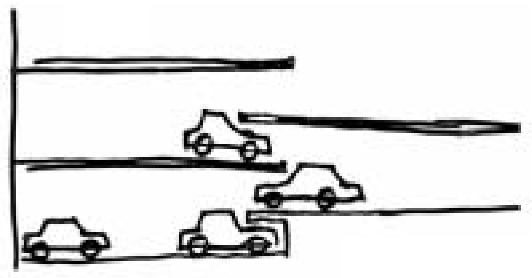
(a)

(b)

Figure 4: Displays for the 'Parking Garage’ problem. (a) section drawing of a split level dwelling by Le Corbusier. (b) zigzag furniture.

In the last stage of the design process, the subject manages to successfully transfer and apply the analogical principle of superposition, from the visual sources to the design problem. Two additional sketches are produced in section (Figures 5a and 5b), illustrating that the design solution is based on the idea of overlapping split-levels between both wings. While sketching the subject comments:

"The problem that I am facing is how to arrange 120 cars in such a way that I could overcome lack of space. I will check if there is a better way to arrange the cars…Yes there is a possibility to do so by overlapping floors between both wings… "



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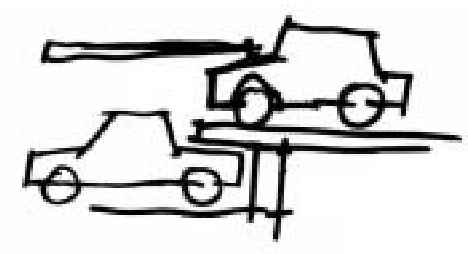

(a)

Figure 5: Sketch III (well-defined problem). (a) Relationship between two overlapping cars in section drawing. (b) Relationship between overlapping cars along the different split-levels in section drawing.

Analogical reasoning is successfully applied. The designer is able to identify, retrieve, and transfer the structural principle from a 'within domain' display, which is crucial for finding the single appropriate solution to the well-defined problem.

Successful ill-defined problem solving aided by visual analogy

In contrast to the previous design session, the student started from a general perusing over the available visual sources. Before focusing on any specific image the subject says:

"I am now looking at the visual displays… perhaps they will help me to focus on the program requirements… and [will also aid] in finding a possible solution for the viewing-terrace [design problem]."

With the purpose of identifying structural relationships that may help to establish an analogy, the student decides to explore some of the visual displays. Suddenly, the subject focuses on a particular 'between-domain' image that seemed to deal with the principle of "digging into the ground", and comments:

"Now looking at the graphic information displayed here I see that the spiral [figure 6]…helps me to think about [the principle of] digging into the ground…"

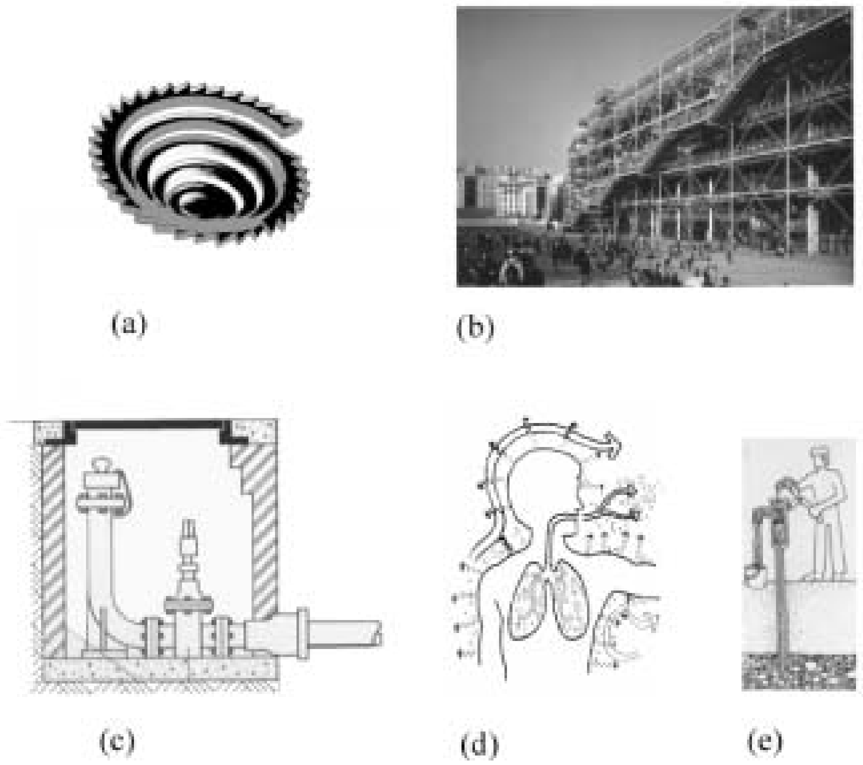

Figure 6: Displays for the 'Viewing Terrace’ problem. (a) downwards spiral object. (b) Pompidou Cultural Center by Renzo Piano. (c) fire brigade connection point. (d) respiratory and perspiration systems. (e) water spreading from a canilla.

The novice designer continues working, and establishes a mapping of ‘deep’ (structural) relationships between the visual source and part of the design requirements. In the next stage of the design process, the subject succeeds in transferring them to the problem. A sketch is made illustrating the relationship between the precipice and a sector of the viewing terrace that have a strong contact with the ground (section, figure 7).

“If I want to design a part of the viewing terrace with maximum contact with the ground… I need to work in drawing section. The question is how to design the viewing-terrace to answer to the design requirements. Well… there might be many possibilities…In order to reach a maximum contact I can dig into the ground through a tunnel, or through a canal… This is a part [of the viewing-terrace] going deep into the ground…It is clear that this part is surrounded all over by ground so that [the terrace] is in maximum contact with it [the ground].”

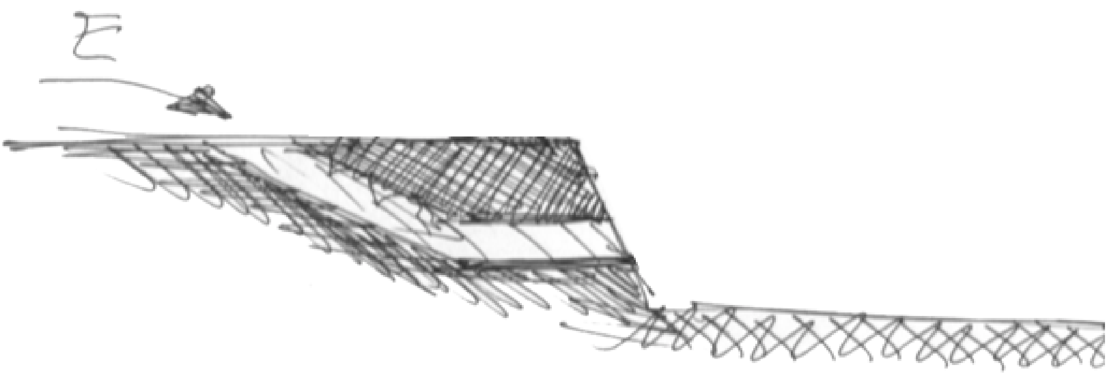

Figure 7: Sketch I (ill-defined problem). Section drawing of the viewing terrace -maximum contact with the ground.

The subject continues working, and in an attempt to identify an analogical principle that might help to deal with the second programmatic requirement of “minimum contact with the ground”, the board is surveyed once again. While exploring the functions of ‘within domain’ and ‘between domain’ images, principles such as ‘climbing up’, ‘exit out’, and ‘detachment’ are noticed, and then used to establish an additional analogy with the design problem:

“I can see the facade of the Pompidou [Cultural Center] [figure 6b] which makes me feel a sort of climbing up. Oh but image 17 [figure 6c], image 9[figure 6d], and the water spreading out [figure 6e] help me to understand the idea of exit out or detachment from the ground.”

The designer continues developing the initial sketch, and manages to transfer the above-mentioned analogical principles to the design problem. The underground sector of the viewing terrace is successfully connected with a new sector largely detached from the ground (see section and plan, figure 8).

“Now I will try to deal with the second design constraint concerned with the design of the remaining part of the viewing-terrace with a minimum contact [with the ground]. It is impossible to totally detach the viewing terrace from the ground, at least with the available technology of today…Therefore I would think in some kind of technology that might allow me to suspend part of the viewing terrace on the air. I will add a couple of columns so that the suspended plate will hang on them, while it is connected to the underground passage through this connecting point [staircases].”

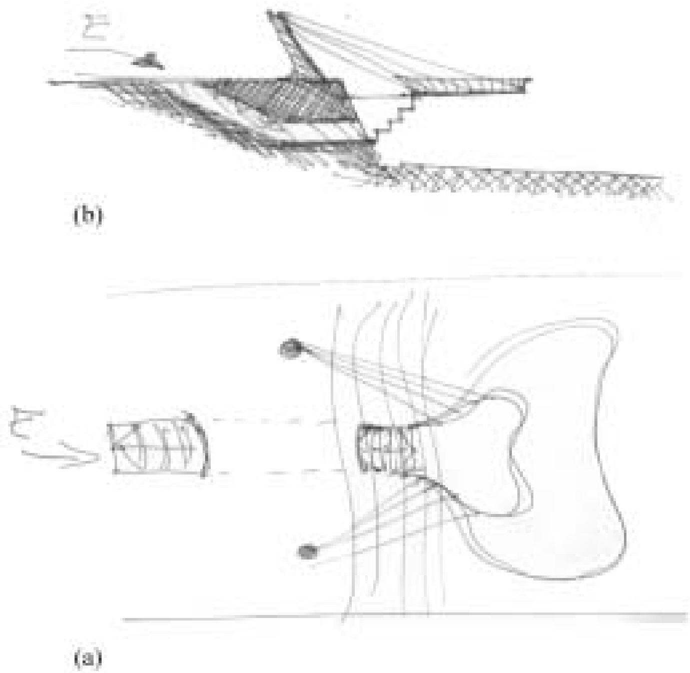

Figure 8: Sketch I (ill-defined problem). (a) Plan and (b) Section drawings of the viewing terrace – maximum and minimum contact with the ground.

In the final stage of the design process, the novice student was able to map and transfer ‘deep’ relationships between the ‘viewing-terrace’ problem, and ‘suspension’ and ‘underground’ principles from different analogical sources. Although a number of different designs were possible, we maintain that the availability of visual displays, and instructions to use analogy aided the subject in finding a successful solution. As a result the programmatic design requirements, of keeping each sector of the ‘viewing terrace’ with maximum and minimum contact with the ground were fully met.

By comparing the two design sessions, we found a number of differences in the way subjects approach the problem in each design context. When an ill-defined problem is assigned, the designer starts the session by familiarizing himself with the assortment of visual displays. The student is confident that these will help to clarify the design goals. Although both a 'within domain' visual display and 'between domain' visual displays are identified as potential analogs, the latter seemed to be more helpful in finding a successful design solution. In ill-defined problem solving, where a large number of unpredictable solutions is possible to be found, 'between domain' (remote) visual displays proved to have better chances to contribute in finding novel design solutions. The production of sketches were helpful to represent the way that the analogical principle is applied to the design solution.

Contrary to this, when a well-defined problem is assigned, the other student begins the session with an attempt to clarify what the main goals of the problem are. Although well-defined problems are characterized by completely specified initial conditions, as well as clear goals, the lack of experience does not allow the novice designer, in the first stage of the design process, to understand what the problem constraints are. The design session turned out to be a 'puzzle-like' solving problem, which called for the use of an efficient algorithm to generate a satisfactory solution. But the inexperienced student, who has not yet developed knowledge structures, was unable to apply any algorithm or some kind of routine procedure. The first sketches aided to visualize and understand the major constraints, and only afterwards to start looking at the visual displays as potential analogs. Although the student was also able to identify a 'between domain' display and a 'within domain' display both as potential analogs, the latter (a display that belongs to a domain close to the problem at hand) helped to establish a 'deep' analogy that lead to the unique design solution.

Well-defined and ill-defined design problems: quantitative results

In this section we show statistical results regarding the use of visual analogy obtained in the different groups of subjects that participated in the empirical tasks. In order to test the hypothesis of this work, we carried out the experiment described above. The individual performance of students and professional designers in the test and control conditions, were compared in both design problems. In the ill-defined problems/ test condition, 68 solutions were obtained (21 by architects, 25 by advanced designers, and 22 by beginning designers). In the ill-defined problems/ control condition, 62 solutions were obtained (19 by architects, 22 by advanced designers, and 21 by beginning designers). In the well-defined problems/ test condition, 38 solutions were obtained (11 by architects, 13 by advanced designers, and 14 by beginning designers). In the well-defined problems/ control condition, 35 solutions were obtained (11 by architects, 12 by advanced designers, and 12 by beginning designers).

The hypothesis, that the use of visual analogy plays a more important role in ill-defined design problem-solving than in well-defined problem-solving, was partially confirmed. Contrary to what was predicted with the provision of visual displays and explicit instructions to use analogy, students who solved ill-defined design problems performed *as good as* those who solved well-defined design problems. However, architects who solved ill-defined design problems achieved significantly better results, than those architects who solved well-defined design problems. Tables 1 and 2 present results of a comparison between test and control conditions in both design contexts.

A discussion and main conclusions about these findings are offered in the next section.

Experimental Condition	*Displays Provided Instructions to use Analogy (Control Condition)*	*Displays Provided No instructions to use Analogy (Test Condition)*	
		p	*phi*
Beginning Students	---------------	<.063	.387
Advanced Students	---------------	<.042	.415
Architects	---------------	<.148	.313

Table 1: Well-defined design problems: quality of designs as a result of the use of visual analogy.

Experimental Condition	*Displays Provided Instructions to use Analogy (Control Condition)*	*Displays Provided No instructions to use Analogy (Test Condition)*		
	Mean	*p*	*t*	
Mean				
Beginning Students	2.621	<.001	-3.10	3.463
Advanced Students	2.939	<.002	-2.98	3.731
Architects	3.236	<.001	-3.68	3.984

Table 2: Ill-defined design problems: quality of designs as a result of the use of visual analogy.

Conclusions and discussion

A comparative analysis of results, obtained in the experiment where visual displays and explicit instructions to use analogy were given, partially validates the working hypothesis, which states that: the use of analogy plays a more important role in ill-defined design problem solving than in well-defined problem solving.

From results of this experiment we see that inexperienced students, who have not yet developed knowledge structures, have the cognitive ability to use analogy as a problem-solving strategy. The use of analogy for them, has a similar importance in ill-defined problem-solving (when trying to find unexpected relations between certain sources and the problem), as in well-defined problem-solving (when looking for a unique specific analogical relation). This contradicts the expectation that students who solved ill-defined problems will perform better than those who solved well-defined ones. We suggest that instructions to use analogy contributed to an increase in capturing their attention to previously overlooked structural relationships between, some of the visual sources and the design problem components. It also helped them to enhance the exploration of several unexpected possible solutions. Although we thought that students are able to successfully use visual analogy in ill-defined problem solving, we also thought that inexperienced students, who generally lack problem-solving algorithms, are not able to spontaneously apply routine processes to successfully solve well-defined problems. One of the reasons for thinking in this way is that searching for a finite number of possible solutions, which are supposed to be reached through the use of an appropriate algorithm, is an unfamiliar and non-routine process for novices. Moreover, the use of analogy is limited to establishing high-level relations between visual sources and the target problem. Therefore, students who generally do not have a large knowledge base relevant to the problem, usually have difficulty to identify a single solution principle. However, it is seen that the implementation of analogy might have been especially useful in providing a new understanding of an unknown domain in terms of a known domain, and thus assisted them in well-defined problem solving as well.

Results from the group of architects validated the research hypothesis. It is observed that these subjects benefited from the use of analogy, which helped to enhance the quality of their design solutions in ill-defined problem-solving, but did not benefit from the use of analogy in solving well-defined problems. It can be said, that instructions to use visual analogy encouraged architects to expand the boundaries of known and even conventional ill-defined designs, while searching for a number of unpredictable solutions. Reasoning by analogy probably helped them to increase their awareness regarding unexpected 'deep' relations between certain clue-harboring displays and the design target. This might lead them to new reformulations of ill-defined problems in non-conventional ways. However, in a comparative analysis of well-defined problems, it was seen that instructions to use visual analogy did not lead to different results in the test condition when compared to the control condition. An interpretation of these findings is that architects have enough expertise to successfully identify, and use, potential analogical sources, and thus do not need additional instructions to reason by analogy. Another explanation is that experienced architects have such developed knowledge structures, that the use of analogy in well-defined problems cannot assist them further. Therefore, it is possible, that at least for conceptual design problem-solving, instead of using analogy, architects preferred to retrieve familiar algorithms, routine processes, or adapt existing solutions, which led them to successful well-defined problem solutions. If this argument is valid, it can be concluded that mastering problem-solving algorithms and adapting existing solutions to similar problems, might be a better strategy for well-defined design problem-solving.

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